

SPRINGBOARD 6 LESSON 27 PROBLEM SOLVING 6

TOTAL TIME

45
MINUTES

Objectives:

- Solve mathematical problems or puzzles, recognise and explain patterns
- Explain methods and reasoning orally and in writing

Vocabulary:

- sequence
- pattern
- multiple
- efficient
- square number

By the end of the lesson the children should be able to:

- generate and extend sequences, identify and describe patterns;
- begin to recognise why some methods are more efficient and use them to solve problems;
- know what to write in the 'Show your method' box.

Resources:

- a counting stick
- Activity Sheet 27.1
- interlocking cubes
- an empty box
- calculators

ORAL AND MENTAL STARTER

5
MINUTES

Using a counting stick count forwards from 0 in steps of 4 to 40.

Repeat, but stopping at intervals, e.g. 24.

Q: What multiplication facts involving 4 and 24 can you tell me?

Collect and record $6 \times 4 = 24$ and $4 \times 6 = 24$.

Q: Now what two division facts can you tell me?

Collect and record $24 \div 4 = 6$ and $24 \div 6 = 4$.

Count in steps of 5 and 6 and collect associated multiplication and division facts.

MAIN TEACHING ACTIVITY

Show the children an empty box which you explain once held interlocking cubes. Say that when using the cubes in the box to make towers of 4 cubes, there was 1 cube left over.

Q: How many cubes could there have been in the box?

Make a tower of 4 cubes and show the 1 cube left over.
Write on the board: $1 \times 4 + 1 = 4 + 1 = 5$ cubes.

Say there were more than 5 cubes in the box.
Make two towers of 4 cubes and show the 1 cube left over.

Q: How many cubes have I got now?

Collect answers and, on the board, record: $4 + 4 + 1 = 9$ cubes.

Q: How else can we record this?

Record: $2 \times 4 + 1 = 8 + 1 = 9$ cubes under the first statement.

Say there were more than 9 cubes in the box. With the children, collect other possibilities for the number of cubes in the box and record then as shown below.

$$1 \times 4 + 1 = 4 + 1 = 5$$

$$2 \times 4 + 1 = 8 + 1 = 9$$

$$3 \times 4 + 1 = 12 + 1 = 13$$

$$4 \times 4 + 1 = 16 + 1 = 17$$

$$5 \times 4 + 1 = 20 + 1 = 21$$

Q: Can you see a pattern for the number of cubes?

Share suggestions.

Q: Can you describe the pattern in words?

Ask the children to write a sentence that describes the pattern. Discuss the children's sentences and agree that the pattern is the 4 times table plus 1.

Say the number of cubes in the box was between 35 and 40. With the children recite the 4 times table and add 1 to generate numbers in the sequence. Stop at 41. Establish that the number of cubes in the box was 37.

Say that in another box of cubes, when you made towers of 5 cubes there were 3 left over, and when you made towers of 6 there was 1 left over.

Q: How many cubes were there in the box?

Discuss the problem.

Q: What multiplication tables should we use?

Establish that it is the 5 and 6 times tables.

Start the sequences with the children, recording on the board:

$$1 \times 5 + 3 = 5 + 3 = 8 \quad 1 \times 6 + 1 = 6 + 1 = 7$$

$$2 \times 5 + 3 = 10 + 3 = 13 \quad 2 \times 6 + 1 = 12 + 1 = 13$$

Agree that 13 cubes is a possible answer.

Say that there were over 40 cubes in the box and ask the children to continue the sequences to find the number of cubes in the box.

Collect the children's solutions and correct any errors or misunderstandings. Agree that the answer is 43 cubes. Emphasise how listing sequences like this is a useful strategy for solving these types of problems.

Give out Activity Sheet 27.1, and calculators.

Ask the children to read the first problem.

Q: Do you recognise the problem? What strategies can you use to solve it?

Establish the problem is the same kind of problem as the cubes and towers.

Discuss the children's strategies and ask them to solve the problem, recording their work in their 'Show your method' box on the Activity Sheet.

PLENARY

Collect children's answers and correct any errors or misunderstandings.

Q: What did you write in the box?

Share contributions and the children's writing.

Q: Where did you start your sequences?

Establish that the children need not have started at:

$$1 \times 5 + 3 = 8 \quad 1 \times 4 + 3 = 7$$

$$2 \times 5 + 3 = 13 \quad 2 \times 4 + 3 = 11$$

Q: What other information is given?

Point out that Hameed made over 55 sweets.

Q: Where could we have started?

Discuss suggestions. Agree that starting at 10×5 and 10×4 would be sensible. With the children, extend the sequences, recording on the board:

$10 \times 5 + 3 = 50 + 3 = 53$	$10 \times 4 + 3 = 40 + 3 = 43$
$11 \times 5 + 3 = 55 + 3 = 58$	$11 \times 4 + 3 = 44 + 3 = 47$
$12 \times 5 + 3 = 60 + 3 = 63$	$12 \times 4 + 3 = 48 + 3 = 51$
$13 \times 5 + 3 = 65 + 3 = 68$	$13 \times 4 + 3 = 52 + 3 = 55$
	$14 \times 4 + 3 = 56 + 3 = 59$
	$15 \times 4 + 3 = 60 + 3 = 63$
	$16 \times 4 + 3 = 64 + 3 = 67$

Q: Which multiple of 5 is close to 55?

Establish that 55 is a multiple of 5.

Q: Which multiple of 4 is close to 55?

Establish that 52 is a multiple of 4.

On the lists on the board underline 11×5 and 13×4 . Say that to save writing and to be more efficient we could have started with these calculations to form the sequences.

Q: What would we have written in the 'Show your method' box?

Discuss the children's suggestions. Emphasise that they should write down all their calculations in this box to help them answer the problem and get the marks.

Ask the children to read the second question.

Q: Is this problem similar to the others?

Establish that it is, and confirm the sequences that represent the number of cards. Ask children to solve the problem, recording their calculations in the 'Show your method' box. Remind them that 21 is not a square number. Establish that 81 is a square number.

Remember:

- Writing out the sequence can help you to see the pattern.
- Use all the information you are given to help you to decide where you might start the sequence.
- Similar problems may be described in different ways – always read the question carefully and compare it with problems you have done before.

**LESSON 27 RELATED TEST QUESTION
2002 TEST B (CALCULATOR PAPER)**

9 Jemma thinks of a number.
She says,
*'Add 3 to my number and then multiply the result by 5
The answer is 35'*


What is Jemma's number?

Riaz thinks of a number.
He says,
*'Halve my number and then add 17
The answer is 23'*

What is Riaz's number?

**LESSON 27 RELATED TEST QUESTION
2002 TEST B (CALCULATOR PAPER)**

13 There are 24 coloured cubes in a box.
Three-quarters of the cubes are red,
four of the cubes are blue
and the rest are green.



How many **green** cubes are in the box?

Show your **method**.
You may get a mark.

One more **blue** cube is put into the box.
What fraction of the cubes in the box are **blue** now?

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
9a	4	
9b	12	

Question	Requirement	Additional Guidance
13a	Award TWO marks for the correct answer of 2 If the answer is incorrect, award ONE mark for evidence of an appropriate method, e.g. $\frac{3}{4}$ of 24 = 18 green = 24 – 18 – 4	<i>Answer need not be obtained for the award of the mark.</i>
13b	$\frac{1}{5}$	<i>Accept equivalent fractions. Do not accept '1 in 5' OR '1 : 5'.</i>

ANALYSIS OF CHILDREN'S ANSWERS

- Half of the children working at level 3 answered part (a) of question 9 correctly, though a significant proportion omitted the question. A common mistake was to give the answer 7, suggesting children knew how to complete the first stage of the calculation. Success rates for part (b) fell to 30 per cent for level 3 children. Again the most common error was to complete the first step and record the answer as 6.
- It was rare for a single mark to be awarded to children for part (a) of question 13. Just over 10 per cent of level 3 children were awarded 2 marks. Finding three-quarters of 24 was the stumbling block for many children and the methods recorded showed that children could not make effective use of their calculator to do this. Some 20 per cent of level 3 children answered part (b) correctly. A common error was 5/24. Children added one to the blue cubes but not to the total number of cubes.

IMPLICATIONS FOR PLANNING

- In the oral and mental starter, children should be asked 'think of a number' type problems involving one step then two steps, supported by discussion on the strategies that the children used. The problems should include all four operations and halving and doubling.
- Children should be taught how to check their solutions to 'think of a number' type problems by applying the rules of the problem to their answer.
- Children should be taught how to use a calculator to find fractions of quantities.
- Plan activities that help children to recognise that when increasing or decreasing part of a quantity, the whole quantity must also be increased or decreased.