Ma 2: Number and algebra programme of study

Mathematics: Higher

Using and applying number and algebra

1 Pupils should be taught to:

Problem solving

a select and use appropriate and efficient techniques and strategies to solve problems of increasing complexity, involving numerical and algebraic manipulation

b identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches

c break down a complex calculation into simpler steps before attempting a solution and justify their choice of methods

d make mental estimates of the answers to calculations; present answers to sensible levels of accuracy; understand how errors are compounded in certain calculations

Communicating

e discuss their work and explain their reasoning using an increasing range of mathematical language and notation

f use a variety of strategies and diagrams for establishing algebraic or graphical representations of a problem and its solution; move from one form of representation to another to get different perspectives on the problem

g present and interpret solutions in the context of the original problem

h use notation and symbols correctly and consistently within a given problem

i examine critically, improve, then justify their choice of mathematical presentation; present a concise, reasoned argument

Reasoning

j explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether a particular case may be generalised further and understand the importance of a counter-example; identify exceptional cases when solving problems

k understand the difference between a practical demonstration and a proof

l show step-by-step deduction in solving a problem; derive proofs using short chains of deductive reasoning

m recognise the significance of stating constraints and assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying the assumptions may have on the solution to a problem.

Numbers and the number system

2 Pupils should be taught to:

Integers

a use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use negative integers both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor decomposition

Powers and roots

b use the terms square, positive square root, negative square root, cube and cube root; use index notation [for example, 8^2, $8^{-\frac{2}{3}}$] and index laws for multiplication and division of integer powers; use standard index form, expressed in conventional notation and on a calculator display

Fractions

c understand equivalent fractions, simplifying a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator

Decimals

d recognise that each terminating decimal is a fraction [for example, $0.137 = \frac{137}{1000}$]; recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals [for example, $\frac{1}{7} = 0.142857142857...$] ; order decimals

Percentages

e understand that 'percentage' means 'number of parts per 100', and interpret percentage as the operator 'so many hundredths of' [for example, 10% means 10 parts per 100 and 15% of Y means $\frac{15}{100} \times Y$]

Ratio

f use ratio notation, including reduction to its simplest form and its various links to fraction notation.

Calculations

3 Pupils should be taught to:

Number operations and the relationships between them

a multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1; find the prime factor decomposition of positive integers; understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal, because division by zero is not defined); multiply and divide by a negative number; use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer, fractional and negative powers; use inverse operations, understanding that the inverse operation of raising a positive number to power n is raising the result of this operation to power $\frac{1}{n}$

b use brackets and the hierarchy of operations

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c calculate a given fraction of a given quantity, expressing the answer as a fraction; express a given number as a fraction of another; add and subtract fractions by writing them with a common denominator; perform short division to convert a simple fraction to a decimal; distinguish between fractions with denominators that have only prime factors of 2 and 5 (which are represented by terminating decimals), and other fractions (which are represented by recurring decimals); convert a recurring decimal to a fraction [for example, 0.142857142857... $= \frac{1}{7}$]

d understand and use unit fractions as multiplicative inverses [for example, by thinking of multiplication by $\frac{1}{5}$ as division by 5, or multiplication by $\frac{6}{7}$ as multiplication by 6 followed by division by 7 (or vice versa)] ; multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction

e convert simple fractions of a whole to percentages of the whole and vice versa; then understand the multiplicative nature of percentages as operators [for example, a 15% increase in value Y, followed by a 15% decrease is calculated as $1.15 \times 0.85 \times Y$] ; calculate an original amount when given the transformed amount after a percentage change; reverse percentage problems [for example, given that a meal in a restaurant costs £36 with VAT at 17.5%, its price before VAT is calculated as £$\frac{36}{1.175}$]

f divide a quantity in a given ratio

Mental methods

g recall integer squares from 2×2 to 15×15 and the corresponding square roots, the cubes of 2, 3, 4, 5 and 10, the fact that $n^0 = 1$ and $n^{-1} = \frac{1}{n}$ for positive integers n [for example, $10^0 = 1$; $9^{-1} = \frac{1}{9}$], the corresponding rule for negative numbers [for example, $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$], $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number n [for example, $25^{\frac{1}{2}} = 5$ and $64^{\frac{1}{3}} = 4$]

h round to a given number of significant figures; develop a range of strategies for mental calculation; derive unknown facts from those they know; convert between ordinary and standard index form representations [for example, $0.1234 = 1.234 \times 10^{-1}$] , converting to standard index form to make sensible estimates for calculations involving multiplication and/or division

Written methods

i use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that in many cases only a fraction can express the exact answer

j solve percentage problems, including percentage increase and decrease [for example, simple interest, VAT, annual rate of inflation]; and reverse percentages

k. represent repeated proportional change using a multiplier raised to a power [for example, compound interest]

l calculate an unknown quantity from quantities that vary in direct or inverse proportion

m calculate with standard index form [for example, $2.4 \times 10^7 \times 5 \times 10^3 = 12 \times 10^{10} = 1.2 \times 10^{11}$, $(2.4 \times 10^7) \div (5 \times 10^3) = 4.8 \times 10^3$]

n use surds and π in exact calculations, without a calculator; rationalise a denominator such as $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Calculator methods

o use calculators effectively and efficiently, knowing how to enter complex calculations; use an extended range of function keys, including trigonometrical and statistical functions relevant across this programme of study

p understand the calculator display, knowing when to interpret the display, when the display has been rounded by the calculator, and not to round during the intermediate steps of a calculation

q use calculators, or written methods, to calculate the upper and lower bounds of calculations, particularly when working with measurements

r use standard index form display and how to enter numbers in standard index form

s use calculators for reverse percentage calculations by doing an appropriate division

t use calculators to explore exponential growth and decay [for example, in science or geography], using a multiplier and the power key.

Solving numerical problems

4 Pupils should be taught to:

a draw on their knowledge of operations and inverse operations (including powers and roots), and of methods of simplification (including factorisation and the use of the commutative, associative and distributive laws of addition, multiplication and factorisation) in order to select and use suitable strategies and techniques to solve problems and word problems, including those involving ratio and proportion, repeated proportional change, fractions, percentages and reverse percentages, inverse proportion, surds, measures and conversion between measures, and compound measures defined within a particular situation

b check and estimate answers to problems; select and justify appropriate degrees of accuracy for answers to problems; recognise limitations on the accuracy of data and measurements.

Equations, formulae and identities

5 Pupils should be taught to:

Use of symbols

a distinguish the different roles played by letter symbols in algebra, using the correct notational conventions for multiplying or dividing by a given number, and knowing that letter symbols represent definite unknown numbers in equations [for example, $x^2 + 1 = 82$], defined quantities or variables in formula [for example, $V = IR$], general, unspecified and independent numbers in identities [for example, $(x + 1)^2 = x^2 + 2x + 1$ for all x], and in functions they define new expressions or quantities by referring to known quantities [for example, $y = 2 - 7x$; $f(x) = x^3$; $y = \frac{1}{x}$ with $x \neq 0$]

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b understand that the transformation of algebraic entities obeys and generalises the well-defined rules of generalised arithmetic [for example, $a(b + c) = ab + ac$] ; expand the product of two linear expressions [for example, $(x + 1)(x + 2) = x^2 + 3x + 2$]; manipulate algebraic expressions by collecting like terms, multiplying a single term over a bracket, taking out common factors [for example, $9x - 3 = 3(3x - 1)$] , factorising quadratic expressions including the difference of two squares [for example, $x^2 - 9 = (x + 3)(x - 3)$] and cancelling common factors in rational expressions [for example, $2(x + 1)^2 / (x + 1) = 2(x + 1)$]

c know the meaning of and use the words 'equation', 'formula', 'identity' and 'expression'

Index notation

d use index notation for simple integer powers, and simple instances of index laws [for example, $x^3 \times x^2 = x^5$; $x^2 / x^3 = x^{-1}$; $(x^2)^3 = x^6$]; substitute positive and negative numbers into expressions such as $3x^2 + 4$ and $2x^3$

Equations

e set up simple equations [for example, find the angle a in a triangle with angles a, $a + 10$, $a + 20$] ; solve simple equations [for example, $5x = 7$; $11 - 4x = 2$; $3(2x + 1) = 8$; $2(1 - x) = 6(2 + x)$; $4x^2 = 49$; $3 = \frac{12}{x}$] by using inverse operations or by transforming both sides in the same way

Linear equations

f solve linear equations in one unknown, with integer or fractional coefficients, in which the unknown appears on either side or on both sides of the equation; solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution

Formulae

g use formulae from mathematics and other subjects [for example, for area of a triangle or a parallelogram, area enclosed by a circle, volume of a prism, volume of a cone] ; substitute numbers into a formula; change the subject of a formula, including cases where the subject occurs twice, or where a power of the subject appears [for example, find r given that $A = \pi r^2$, find x given $y = mx + c$] ; generate a formula [for example, find the perimeter of a rectangle given its area A and the length l of one side]

Direct and inverse proportion

h set up and use equations to solve word and other problems involving direct proportion or inverse proportion [for example, $y \propto x$, $y \propto x^2$, $y \propto \frac{1}{x}$, $y \propto \frac{1}{x^2}$] and relate algebraic solutions to graphical representation of the equations

Simultaneous linear equations

i find the exact solution of two simultaneous equations in two unknowns by eliminating a variable, and interpret the equations as lines and their common solution as the point of intersection

j solve simple linear inequalities in one variable, and represent the solution set on a number line; solve several linear inequalities in two variables and find the solution set

Quadratic equations

k solve quadratic equations by factorisation, completing the square and using the quadratic formula

Simultaneous linear and quadratic equations

l solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns, one of which is linear in each unknown, and the other is linear in one unknown and quadratic in the other [for example, solve the simultaneous equations $y = 11x - 2$ and $y = 5x^2$] , or where the second is of the form $x^2 + y^2 = r^2$

Numerical methods

m use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them [for example, $x^3 - x = 900$].

Sequences, functions and graphs

6 Pupils should be taught to:

Sequences

a generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers); generate terms of a sequence using term-to-term and position-to-term definitions of the sequence; use linear expressions to describe the *n* th term of an arithmetic sequence, justifying its form by reference to the activity or context from which it was generated

Graphs of linear functions

b use conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for m and c) that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane; plot graphs of functions in which y is given explicitly in terms of x (as in $y = 2x + 3$), or implicitly (as in $x + y = 7$)

c find the gradient of lines given by equations of the form $y = mx + c$ (when values are given for m and c); understand that the form $y = mx + c$ represents a straight line and that m is the gradient of the line, and c is the value of the y-intercept; explore the gradients of parallel lines and lines perpendicular to these lines [for example, know that the lines represented by the equations $y = -5x$ and $y = 3 - 5x$ are parallel, each having gradient (–5) and that the line with equation $y = \frac{x}{5}$ is perpendicular to these lines and has gradient one-fifth]

Interpreting graphical information

d construct linear functions and plot the corresponding graphs arising from real-life problems; discuss and interpret graphs modelling real situations [for example, distance-time graph for a particle moving with constant speed, the depth of water in a container as it empties, the velocity-time graph for a particle moving with constant acceleration]

Quadratic functions

e generate points and plot graphs of simple quadratic functions [for example, $y = x^2$; $y = 3x^2 + 4$], then more general quadratic functions [for example, $y = x^2 - 2x + 1$] ; find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function; find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions

Other functions

f plot graphs of: simple cubic functions [for example, $y = x^3$] , the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, the exponential function $y = k^x$ for integer values of x and simple positive values of k [for example, $y = 2^x$; $y = (\frac{1}{2})^x$] , the circular functions $y = \sin x$ and $y = \cos x$, using a spreadsheet or graph plotter as well as pencil and paper; recognise the characteristic shapes of all these functions

Transformation of functions

g apply to the graph of $y = f(x)$ the transformations $y = f(x) + a$, $y = f(ax)$, $y = f(x + a)$, $y = a\, f(x)$ for linear, quadratic, sine and cosine functions $f(x)$

Loci

h construct the graphs of simple loci, including the circle $x^2 + y^2 = r^2$ for a circle of radius r centred at the origin of coordinates; find graphically the intersection points of a given straight line with this circle and know that this corresponds to solving the two simultaneous equations representing the line and the circle.

Breadth of study

1 During the Key Stage, pupils should be taught the Knowledge, skills and understanding through:

a activities that ensure they become familiar with and confident using standard procedures for the range of calculations appropriate to this level of study

b solving familiar and unfamiliar problems in a range of numerical, algebraic and graphical contexts and in open-ended and closed form

c using standard notations for decimals, fractions, percentages, ratio and indices

d. activities that show how algebra, as an extension of number using symbols, gives precise form to mathematical relationships and calculations

e activities in which they progress from using definitions and short chains of reasoning to understanding and formulating proofs in algebra and geometry

f a sequence of practical activities that address increasingly demanding statistical problems in which they draw inferences from data and consider the uses of statistics in society

g choosing appropriate ICT tools and using these to solve numerical and graphical problems, to represent and manipulate geometrical configurations and to present and analyse data.